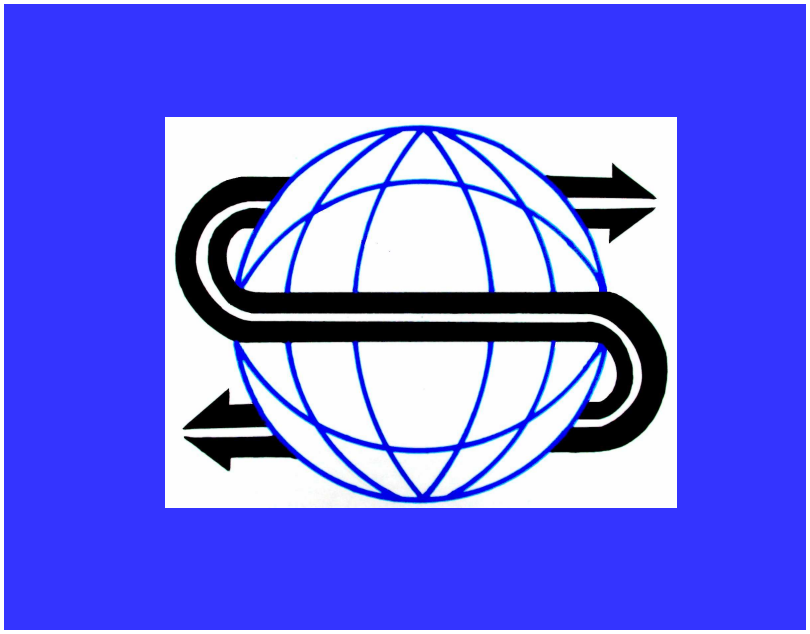

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An XML-based schema for stochastic programs

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Outline

- Motivation and review
- Dynamic and stochastic structure
- A four-stage investment problem
- Instance representations
- OSiL format
- Conclusions and future work

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Why a standard?

- Benchmarking
- Archiving
- Algorithm development
- Distributed computing
- Sharing of problem instances

Why XML?

- Easy to accommodate new features
- Existing parsers to check syntax
- Easy to generate automatically
- Trade-off between verbosity and human readability
- Arbitrary precision and name space
- Stochastic extensions for dynamic and stochastic structure

Stochastic programs

$$
\begin{array}{lll}
\text{“min”} & f_0(x_0) + f_1(x_0, x_1) + \mathrm{K} + f_T(x_0, x_1, \mathrm{K}\ , x_T) & \\
\text{s.t.} & G_0(x_0) & \sim b_0 \\
 & R_1(x_0) & \Delta\ r_1 \\
 & G_1(x_0, x_1) & \Delta\ b_1 \\
 & \mathrm{M} & \mathrm{M} \\
 & G_T(x_0, x_1, \mathrm{K}\ , x_T) & \Delta\ b_T
\end{array}
$$

$$l_0 \le x_0 \le u_0$$

$$l_t \le x_t \le u_t, t = 1, \mathrm{K}\ , T$$

Any data item with nonzero subscript may be random (including dimensions where mathematically sensible)
~ stands for arbitrary relation (≤, =, ≥)

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Constraints involving random elements

$$G_t(x_0, x_1, \mathrm{K}, x_t) \,\Delta\, b_t$$

Δ means ~ with probability 1

or with probability at least β

or with expected violation at most v

or …

Problem classes

- Recourse problems
  - All constraints hold with probability 1
- Chance-constrained problems
  - Typically single stage
- Hybrid problems
  - Recourse problems including features such as chance constraints or integrated chance constraints
- Distribution problems
  - Determine distribution of optimum objective and/or decisions

Example (Birge)

$$\max \sum_{s=1}^{S} p_s (w_s - \beta u_s)$$

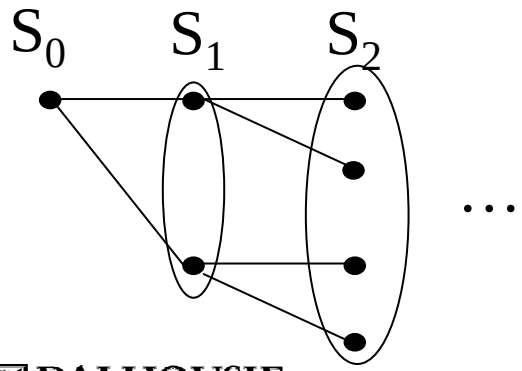
$$\text{s.t.} \quad \sum_{i=1}^{I} x_{0i} = B$$

$$\sum_{i=1}^{I} \alpha_{0is} x_{0i} - \sum_{i=1}^{I} x_{1is} = 0, s \in S_1$$

$$\sum_{i=1}^{I} \alpha_{t-1,i,s} x_{t-1,i,a(s)} - \sum_{i=1}^{I} x_{tis} = 0, s \in S_t, t = 2, \mathrm{K}, T-1$$

$$\sum_{i=1}^{I} \alpha_{T-1,i,s} x_{T-1,i,a(s)} + u_s - w_s = R, s \in S_T$$

$$x_{tis}, u_s, w_s \geq 0$$



$I = 2,\ T = 3,\ B = 55,\ R = 80,$

$\alpha_{t1} = \{1.25, 1.06\},$

$\alpha_{t2} = \{1.14, 1.12\}$

Dynamic and stochastic structure

- Dynamic structure
  - Periods/stages
- Stochastic structure
  - Nonanticipativity
  - Independent random variables
  - Period-to-period independence
  - Scenario tree
  - Factor models
  - ARMA processes
  - Trap states and stochastic problem dimensions

What is an instance?

- Role and number of constraints, objectives, parameters and variables must be known
- Every parameter's value(s) must be known
- Continuous entities vs. discretization
  - Decision variables
  - Objective and constraints
  - Distribution of random variables
  - Time domain

Instance representation

- Internal representations
- SMPS format
  - Limited precision
  - Limited to linear problems
- Algebraic modelling languages
  - Discrete distributions only
  - Limited consistency checks
- OSiL format

What is a stage?

- Stages form a subset of the time structure
- Stages comprise both decisions and events
- Events must either precede all decisions or follow all decisions
- Should a stage be *decision – event* or *event – decision*?

Why is there a problem?

- AMPL-like declarations:

```
set time ordered;
param demand{t in time} random;
Production_balance {t in time}:
Inv[t-1] + product[t] >= demand[t] + Inv[t];
```

- Is the constraint well-posed?
- At least two possible interpretations
  - Inv[t] set after demand[t] known: recourse form, well-posed
  - Inv[t] set before demand[t] known: undeclared chance constraint

SMPS format

- Three files based on MPS format
  - Core file for deterministic problem components
  - Time file for dynamic structure
  - Stoch file for stochastic structure
- Disadvantages
  - Old technology
  - Limited precision (12 digits, including sign)
  - Limited name space (8 characters)
  - Direction of optimization (min/max) ambiguous
  - Linear constraints, quadratic objective only

Example (SMPS)

$I = 2$, $T = 3$, $B = 55$, $R = 80$, $\alpha_{t1} = \{1.25, 1.06\}$, $\alpha_{t2} = \{1.14, 1.12\}$

Core file

```
ROWS
    Budget0
    Object
    Budget1
    Budget2
    Budget3
COLS
    X01       Budget0    1.0
    X01       Budget1    1.25
    ...
RHS
    rhs1      Budget0    55.
    rhs1      Budget3    80.
ENDATA
```

Stoch file

```
BLOCKS          DISCRETE
 BL Block1                    0.5
    X01        Budget1        1.25
    X02        Budget1        1.14
 BL Block1                    0.5
    X01        Budget1        1.06
    X02        Budget1        1.12
 BL Block2                    0.5
    X11        Budget2        1.25
    X12        Budget2        1.14
    ...
ENDATA
```

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Algebraic modelling languages

- Characteristics
  - Similar to algebraic notation
  - Powerful indexing capability
  - Data verification possible
- Disadvantages
  - Discrete distributions only
  - Limited consistency checks for stochastic structure

AMPL model

```
param T;
param penalty;
param budget;
param target;
set instruments;
set scenarios;
param prob{scenarios};
set slice{t in 0..T} within scenarios;
param ancestor {t in 1..T, s in slice[t]};
var over {slice[T]};
var under{slice[T]};
param return {t in 1..T,  i in instruments,s in slice[t]};
var   invest {t in 0..T-1,i in instruments,s in slice[t]};

maximize net_profit:
   sum{s in scenarios} prob[s]*(over[s] - penalty*under[s]);

subject to wealth{t in 0..T, s in slice[t]}:
(if t < T then sum{i in instruments} invest[t,i,s]) =
(if t = 0 then budget
          else sum {i in instruments}
                     return[t,i,s]*invest[t-1,i,ancestor[t,s]]
          + if t = T then under[s] - over[s] + target);
```


OSiL Schema

- Written in XML
- Very flexible
- Intended to handle as many types of mathematical programs as possible
  - Linear and integer
  - Nonlinear
  - Stochastic
  - …

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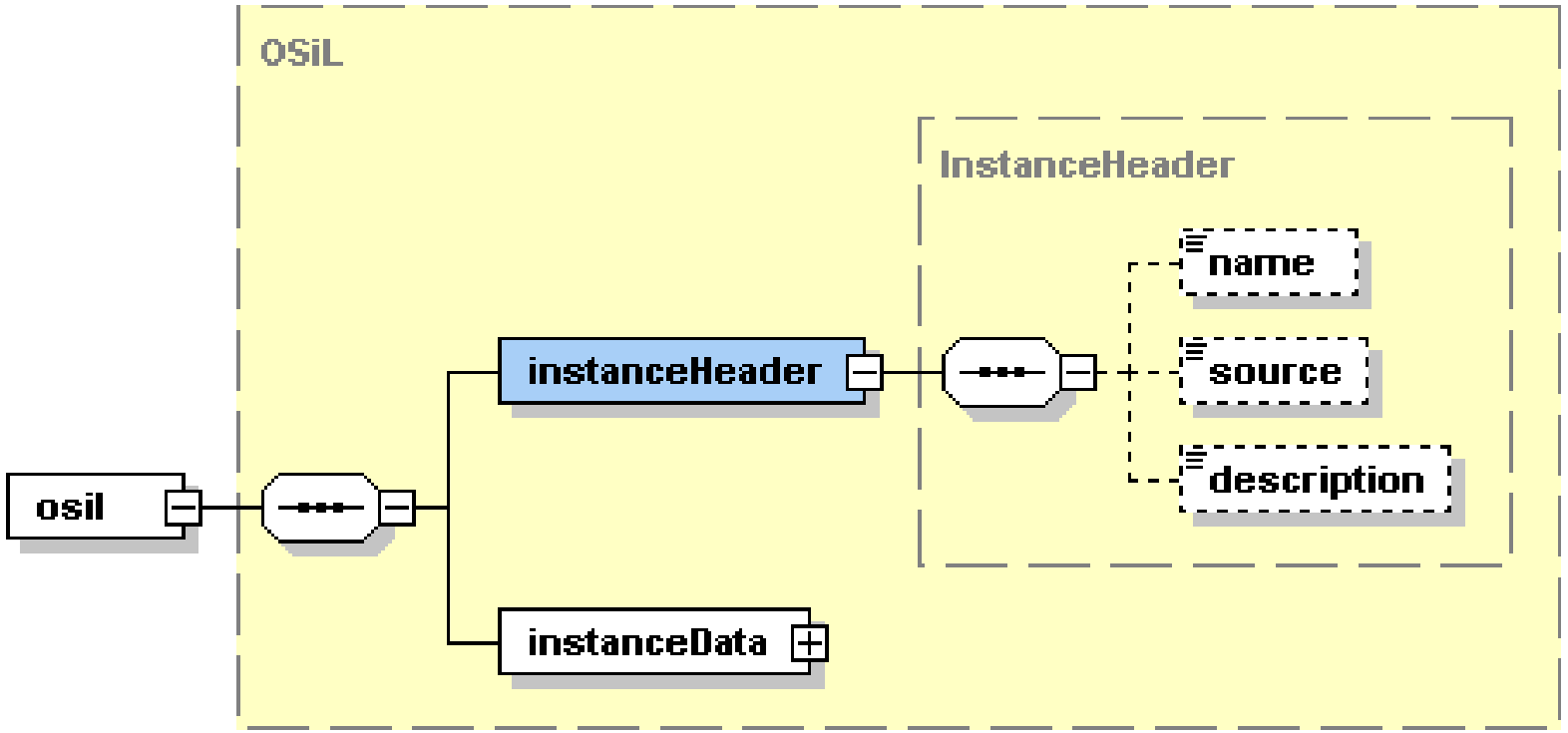
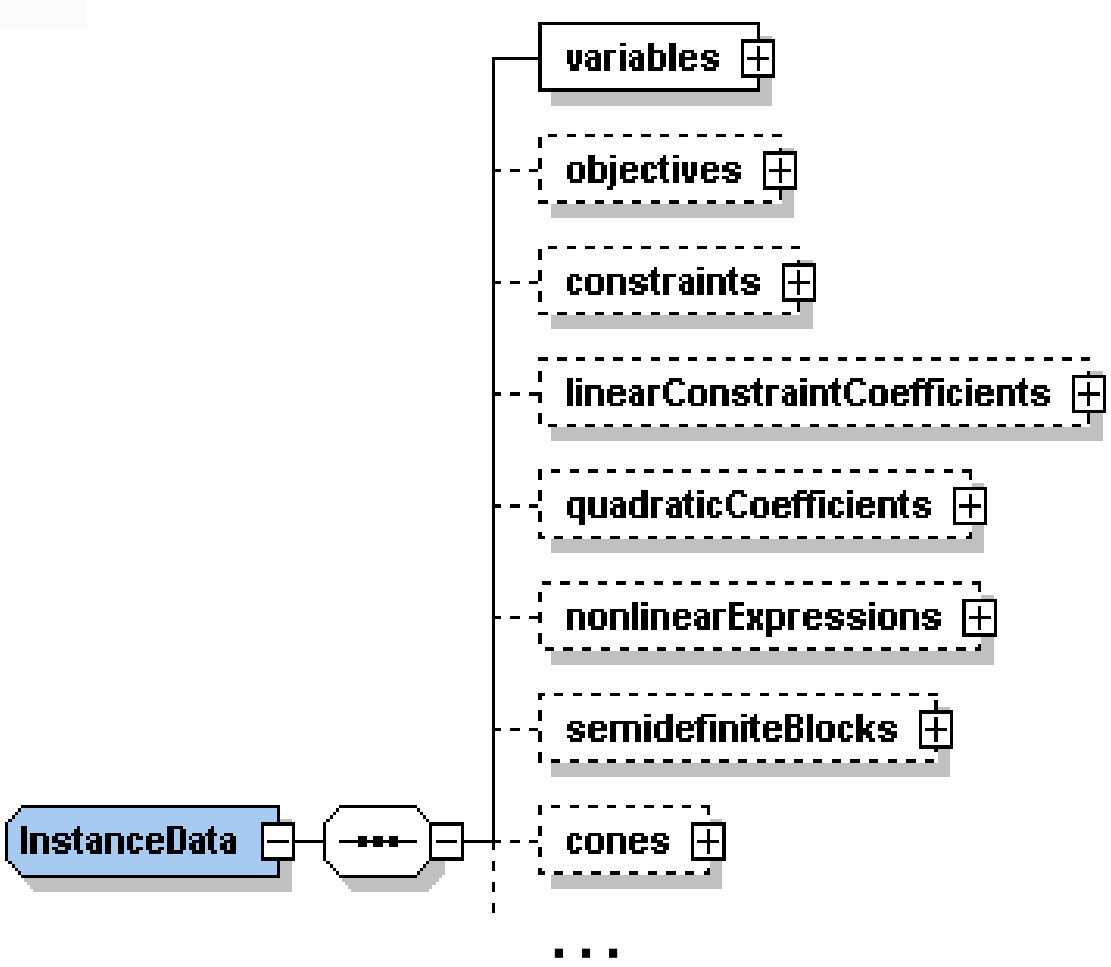
OSiL Schema – Header information



Header information – Example

```
<?xmlversion="1.0"encoding="UTF8"?>
  <osil xmlns="os.optimizationservices.org"
    xmlns:xsi=http://www.w3.org/2001/XMLSchemainstance
    xsi:schemaLocation="OSiL.xsd">
   <instanceHeader>
     <name>FinancialPlan_JohnBirge</name>
     <source>
        Birge and Louveaux, Stochastic Programming
     </source>
     <description>
        Three-stage stochastic investment problem
     </description>
   </instanceHeader >
   <instanceData>
     ...
   </instanceData>
  </osil>
```


OSiL Schema – Deterministic data



Instance data – Constraints, objectives, variables

```
<variables numberOfVariables="8">
   <var name="invest01" type="C" lb="0.0"/>
   <var name="invest02"/>
   <var name="invest11"/>
   <var name="invest12"/>
   <var name="invest21"/>
   <var name="invest22"/>
   <var name="w"/>
   <var name="u"/>
</variables>
<objectives numberOfObjectives="1">
   <obj maxOrMin="max" numberOfObjCoef= "2" lb="0.0">
      <coef idx="6"/>1.</coef>
      <coef idx="7"/>-4.</coef>
   </obj>
</objectives>
<constraints numberOfConstraints="4">
   <con name="budget0" lb="55" ub="55"/>
   <con name="budget1" lb="0" ub="0"/>
   <con name="budget2" lb="0" ub="0"/>
   <con name="budget3" lb="80" ub="80"/>
</constraints>
```


Instance data – Core matrix (sparse matrix form)

```
<linearConstraintCoefficients
        numberOfValues="14">
  <start>
    <el>0</el>
    <el>2</el>
    <el>4</el>
    <el>6</el>
    <el>8</el>
    <el>10</el>
    <el>12</el>
    <el>13</el>
    <el>14</el>
  </start>
```

```
<rowIdx>
  <el>0</el>
  <el>1</el>
  <el>0</el>
  <el>1</el>
  <el>1</el>
  <el>2</el>
  <el>1</el>
  <el>2</el>
  <el>2</el>
  <el>3</el>
  <el>2</el>
  <el>3</el>
  <el>3</el>
  <el>3</el>
</rowIdx>
```

```
<value>
  <el>1</el>
  <el>1.25</el>
  <el>1</el>
  <el>1.14</el>
  <el>1</el>
  <el>1.25</el>
  <el>1</el>
  <el>1.14</el>
  <el>1</el>
  <el>1.25</el>
  <el>1</el>
  <el>1.14</el>
  <el>1</el>
  <el>-1</el>
</value>
```

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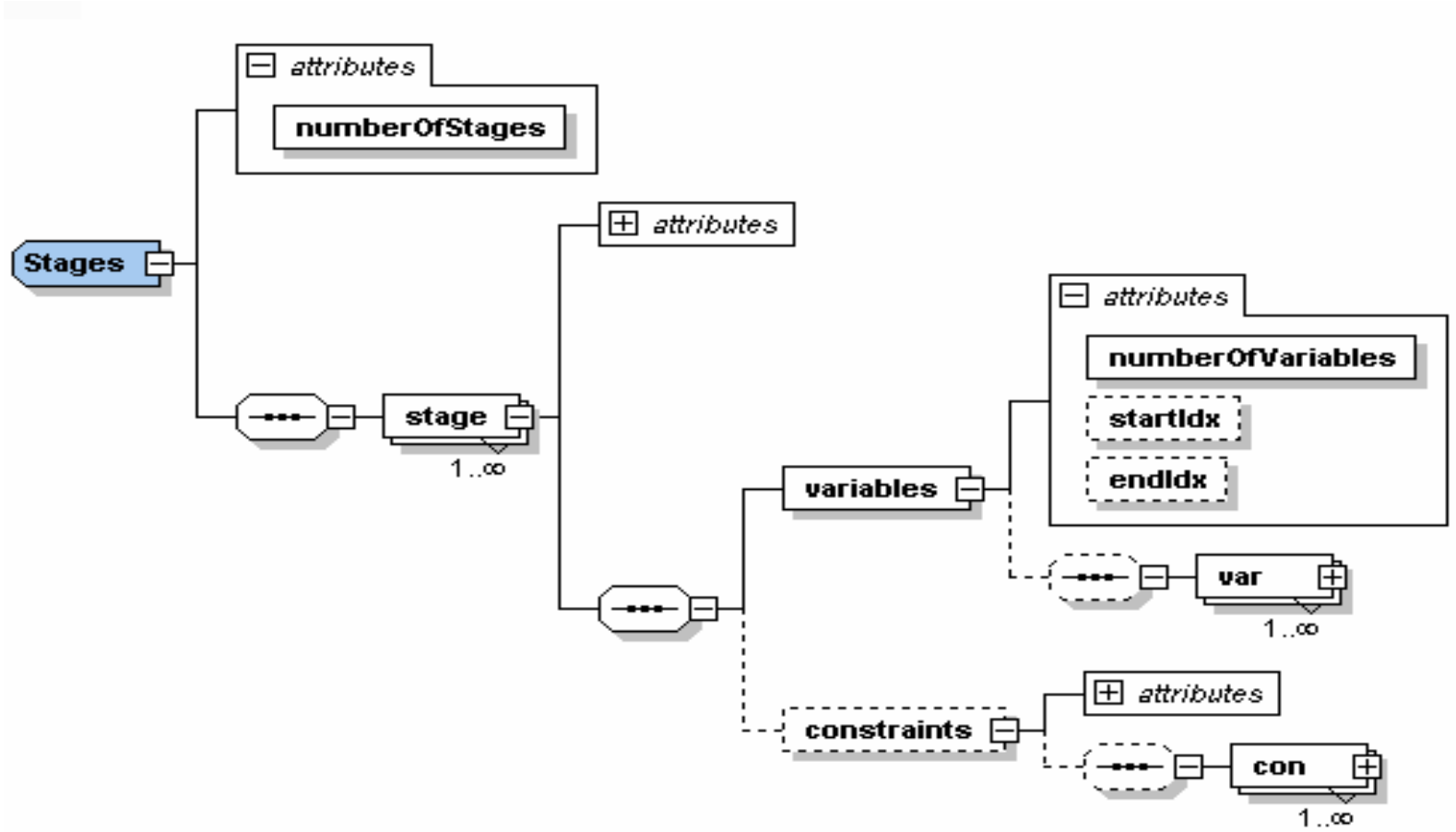
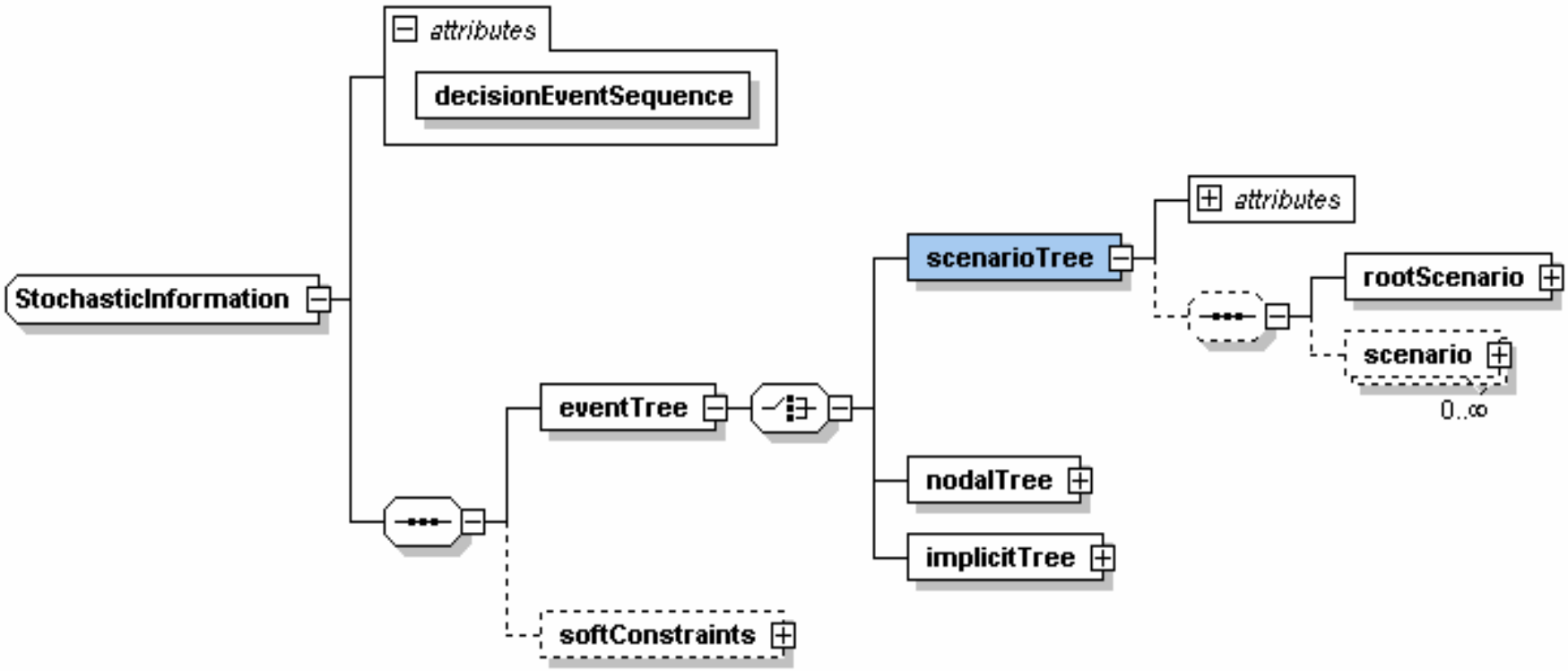
OSiL Schema – Dynamic structure



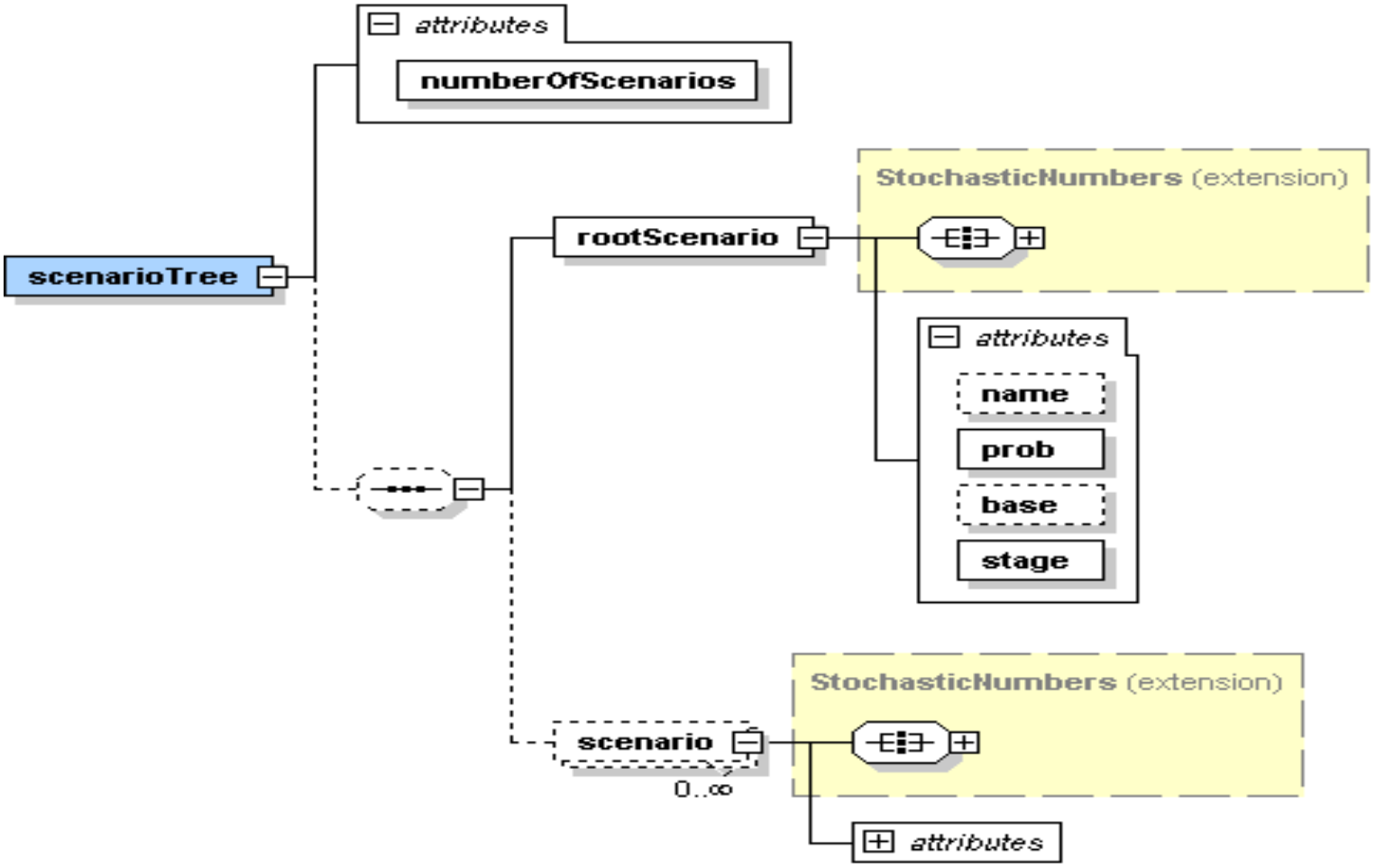
Dynamic information – Example

```
<stages numberOfStages="4">
   <stage>
      <variables numberOfVariables="2" startIdx="0" endIdx="1"/>
      <constraints numberOfConstraints="1" startIdx="0"/>
   </stage>
   <stage>
      <variables numberOfVariables="2" startIdx="2" endIdx="3"/>
      <constraints numberOfConstraints="1" startIdx="1"/>
   </stage>
   <stage>
      <variables numberOfVariables="2" startIdx="4" endIdx="5"/>
      <constraints numberOfConstraints="1" startIdx="2"/>
   </stage>
   <stage>
      <variables numberOfVariables="2">
         <var idx="6">     <var idx="7">
      </variables>
      <constraints numberOfConstraints="1" startIdx="3"/>
   </stage>
</stages>
```


Explicit and implicit event trees



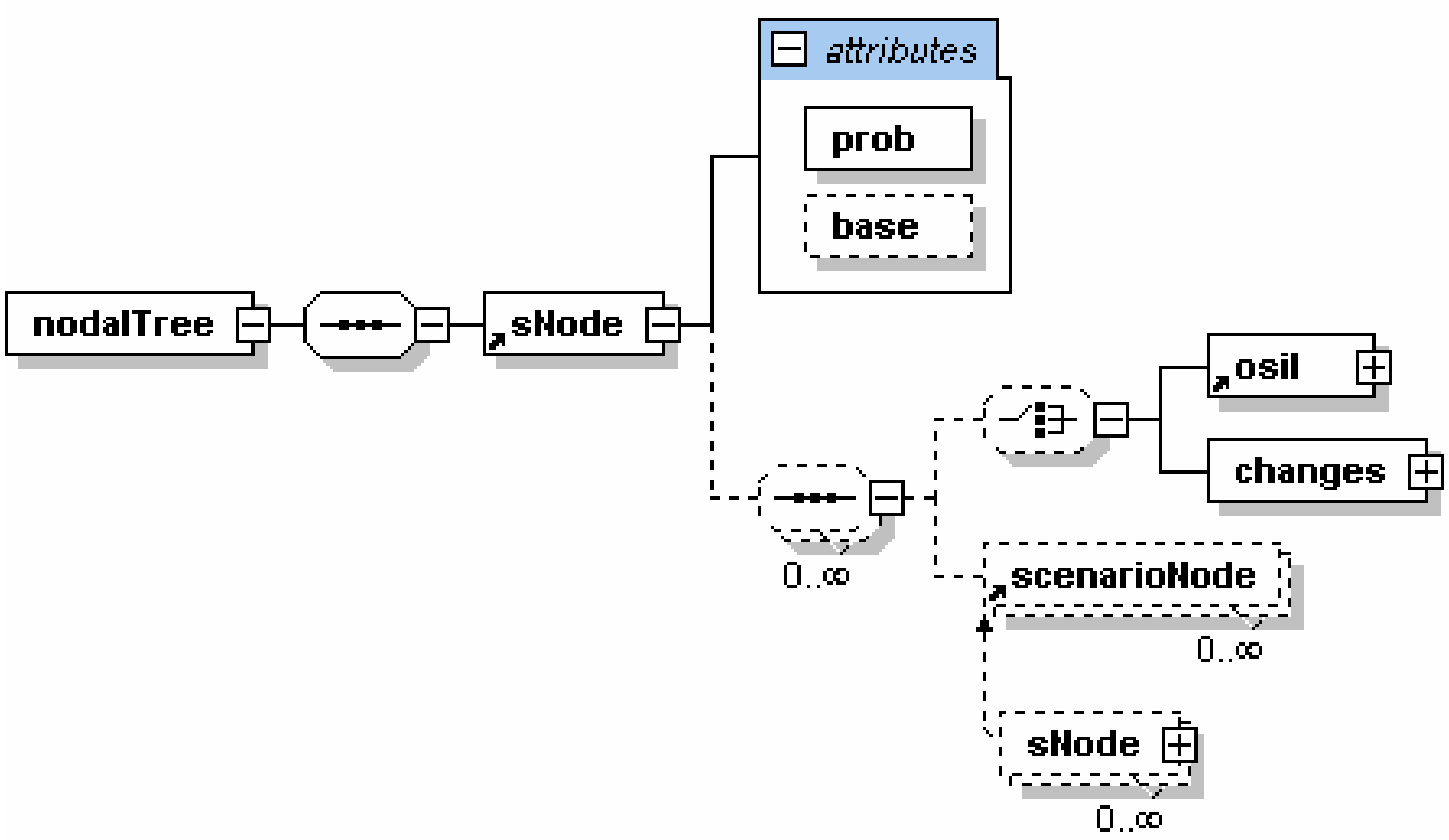
Scenario trees



Scenario tree – Example

```
<stochasticInformation
        decisionEvenSequence="DecisionAfterEvent">
   <eventTree>
      <scenarioTree numberOfScenarios="8">
         <rootScenario prob="1" stage="0"/>
         <scenario prob="0.5" stage="3" parent="0">
            <linearConstraintCoefficients>
               <el rowIdx="3" colIdx="4">1.06</el>
               <el rowIdx="3" colIdx="5">1.12</el>
            </linearConstraintCoefficients>
         </scenario>
         <scenario prob="0.5" stage="2" parent="0">
          ...
```

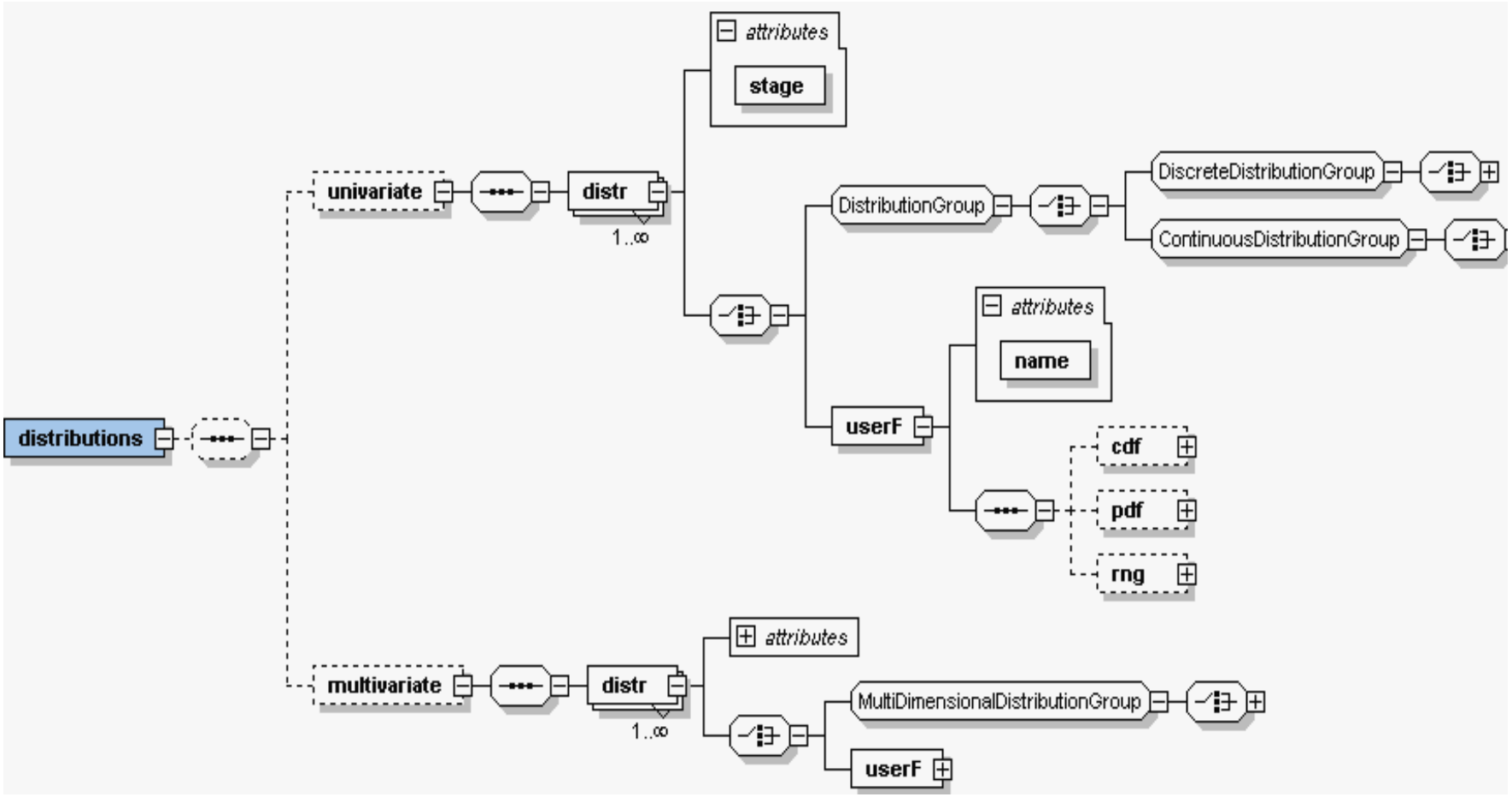

Node-by-node representation for stochastic problem dimensions



Node-by-node – Example

```
<stochasticInformation
        decisionEvenSequence="DecisionAfterEvent">
   <eventTree >
      <nodalTree>
         <sNode prob="1" base="coreProgram">
            <sNode prob="0.5" base="coreProgram">
               <sNode prob="0.5" base="coreProgram">
                  <sNode prob="0.5" base="coreProgram"/>
                  <sNode prob="0.5" base="firstSibling">
                     <changes>
                        <el rowIdx="3" colIdx="4">1.06</el>
                        <el rowIdx="3" colIdx="5">1.12</el>
                     </changes>
                  </sNode>
               </sNode>
               ...
```

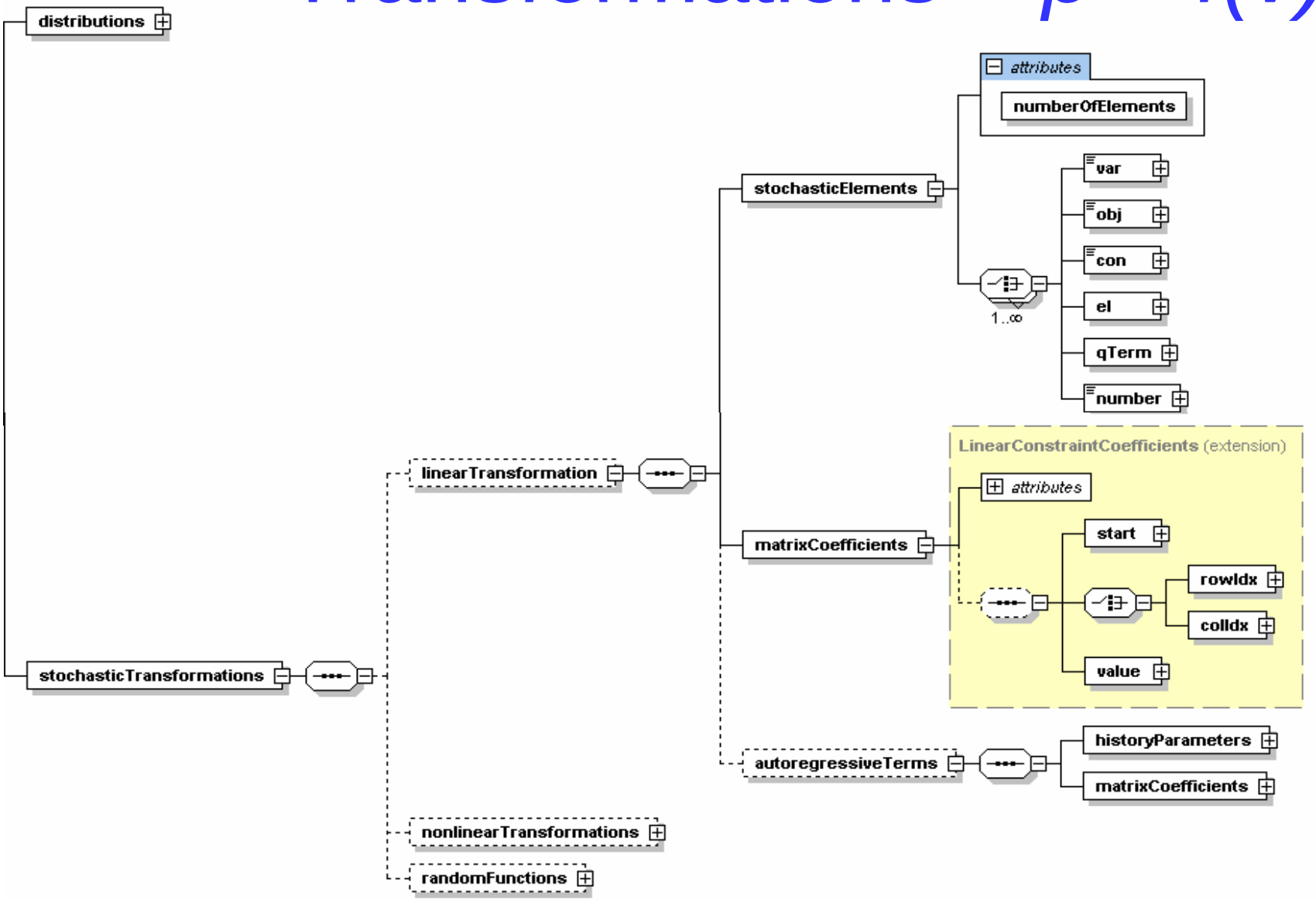

Distributions (implicit tree)



Discrete random vector

```
<distributions>
   <multivariate>
      <distr stage="1">
         <multiDimensionalDistributionGroup>
            <multivariateDiscrete>
               <scenario>
                  <prob>0.5</prob>
                  <el>1.25</el>
                  <el>1.14</el>
               </scenario>
               <scenario>
                  <prob>0.5</prob>
                  <el>1.06</el>
                  <el>1.12</el>
               </scenario>
            </multivariateDiscrete>
         </multiDimensionalDistributionGroup>
      </distr>
       ...
   </multivariate>
</distributions>
```

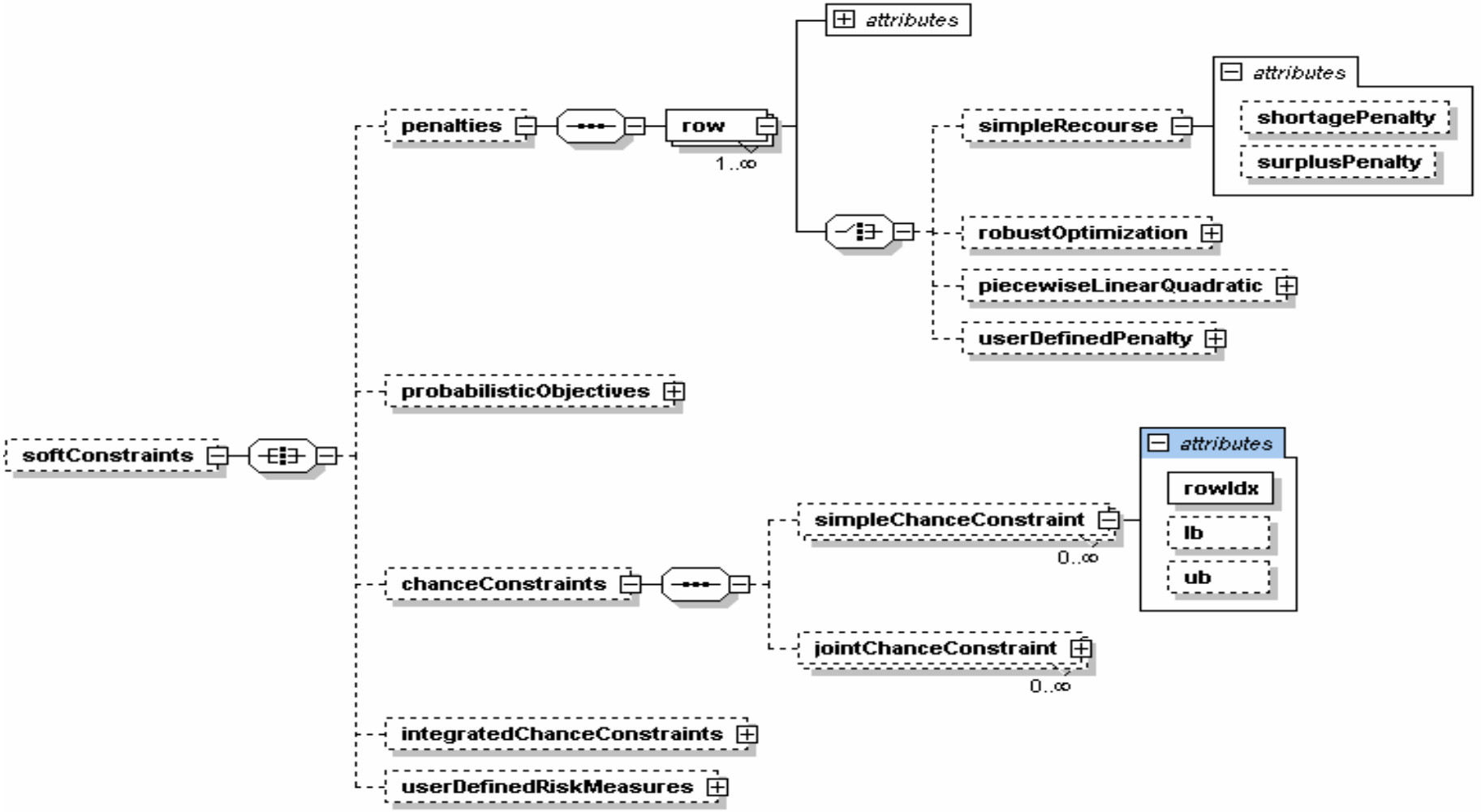

Transformations – $p = f(v)$



Linear transformation – Example

```
<stochasticTransformation>
  <linearTransformation>
    <stochasticElements
        numberOfElements="6">
      <el rowIdx="1" colIdx="0"/>
      <el rowIdx="1" colIdx="1"/>
      <el rowIdx="2" colIdx="2"/>
      <el rowIdx="2" colIdx="3"/>
      <el rowIdx="3" colIdx="4"/>
      <el rowIdx="3" colIdx="5"/>
    </stochasticElements>
    <matrixCoefficients
        numberOfElements="6">
      <start>
        <el>0</el>
        <el>1</el>
        <el>2</el>
        <el>3</el>
        <el>4</el>
        <el>5</el>
        <el>6</el>
      </start>
      <rowIdx>
        <el>0</el>
        <el>1</el>
        <el>2</el>
        <el>3</el>
        <el>4</el>
        <el>1</el>
      </rowIdx>
      <value>
        <el mult="6">1.0</el>
      </value>
    </matrixCoefficients>
  </linearTransformation>
</stochasticTransformation>
```


Penalties and probabilistic constraints



Capabilities

- Arbitrary nonlinear expressions
- Arbitrary distributions
- Scenario trees
- Stochastic problem dimensions
- Simple recourse
- Soft constraints with arbitrary penalties
- Probabilistic constraints
- Arbitrary moment constraints

Nonlinear expression – $(x_0 - x_1^2)^2 + (1 - x_0)^2$

```
<plus>
  <power>
    <minus>
      <var coef="1.0" idx="0"/>
      <power>
        <var coef="1.0" idx="1"/>
        <number value="2"/>
      </power>
    </minus>
    <number value="2"/>
  </power>
  <power>
    <minus>
      <number value="1"/>
      <var coef="1.0" idx="0"/>
    </minus>
    <number value="2"/>
  </power>
</plus>
```

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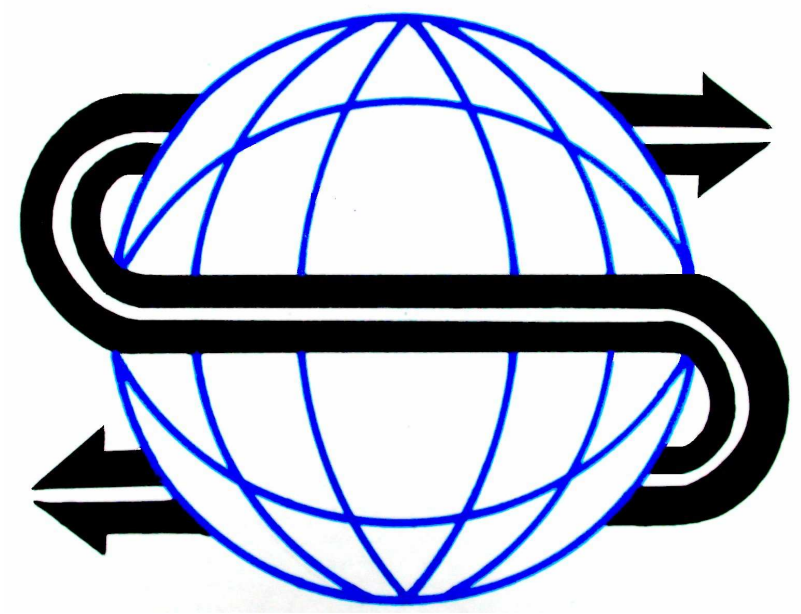
Further work

- Readers
- Internal data structures
- Solver interfaces
- Library of problems
- Buy-in

QUESTIONS?



http://www.optimizationservices.org

http://myweb.dal.ca/gassmann

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